

Secondary School Students' Misconceptions in Algebra Concepts

Chandra Tiwari & Roohi Fatima

Abstract: This paper aims to study the problems in students' understanding and misconceptions in algebra concepts, and their implications on word problems. Cognitive difficulties and conceptual misunderstandings acquired at this stage hamper the learning and interest in Mathematics. The study is significant in the contemporary school scenario in India, especially in teaching and learning of elementary school algebra, and their implications for other topics of secondary school such as probability and trigonometry. This paper is part of a larger project titled "Study of Algebraic Misconceptions of Secondary School in Delhi/NCR. As a sequential exploratory study, it employs a mixed method research, and process of triangulation. Data collection is guided by quantitative method which includes preparation of test instrument. Its validity and reliability are ensured by the investigator before proceeding to refine the test instrument by conducting a pilot study. The data gathered here leads to the qualitative phase which employs case study method to gain insights into thinking process of students, which leads to misconceptions in learning of algebra concepts at secondary stage.

Keywords: Mathematization, Cognitive Difficulties, Secondary Mathematics, Word Problems

Introduction

Mathematics, as a discipline, holds an important place in school curriculum in India. It is an important branch of knowledge, necessary for human growth and to make day-to-day life easy. It develops new concepts and a meaningful symbolic language. The characteristic features of mathematics are abstraction, precision, generality, logic, analysis and systematisation. Mathematics has always been considered as a very important subject in schools and the emphasis on teaching and learning of mathematics has been immense for both the school and parents at home. Apart from contributing to one's personal growth, it is considered a gateway to many lucrative professions in the world. Mathematics education is a relatively new area of study. According to the National Curriculum Framework (NCF), "The main goal of Mathematics Education in schools is Mathematization of child's thinking. Vision of Excellent Mathematical Education is based on twin premises that all students can learn Mathematics and all students should learn Mathematics" (NCERT, 2005). In the Indian context, mathematics

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has been a dreaded subject with a large number of student failures. Taking note of the low level of mathematics learning, it was decided to reform the curriculum following NCF, 2005. The new curriculum was to be child-centred and ensured that the overall learning, including mathematics, would be an enjoyable experience for the students. New textbooks for grades 1-12 following the NCF, 2005 were brought out by the National Council of Educational Research and Training (NCERT) through a collaborative process involving educators and teachers. The NCERT textbooks in mathematics have undergone several important and noticeable changes in teaching approach, particularly in the primary grades. The presentation of algebra has changed considerably following the NCF, 2005. A look at the mathematics textbooks indicate that algebra has changed from what it used to be i.e. learning to simplify algebraic expressions, using algorithms and word problems far removed from the context of the students. There is an effort today to understand the idea of a variable, functional relationships, and the use of letter numbers in different ways. The attempt is to understand algebra as a generalization of many of the ideas that are seen as patterns. One of the issues that remains in the new textbooks is the introduction to symbolic algebra in the middle grades, which follows a largely traditional approach focused on symbol manipulation. Algebra is an important part of the secondary curriculum, bringing mathematics to wider sections of the student population, legitimately requires that more thought be given to how algebra can be dealt and developed in a manner that uses students' prior knowledge.

Algebra in Secondary School Mathematics according to NCF, 2005

At the secondary level, mathematics comprises different topics such as algebra, geometry and probability, but all these topics stand disconnected. The interconnections among the topics are a must for efficient and resourceful teaching. A well-designed comprehensive curriculum helps to construct and integrate important mathematical ideas to build meaningful conceptual structures. The objective of secondary mathematics curriculum is to equip students with important mathematics needed for better educational/ professional/ social choices. It empowers students to investigate, understand, and make meaning of new situations.

Students experience a slow and difficult transition from arithmetic to algebra when they encounter the latter at the upper primary stage. Treading to numerical patterns after numbers, seeing relationship between numbers and forming generalisations lead to the understanding of algebraic identities. Necessity for solving daily life problems using mathematical language leads to introduction of 'variables' or 'unknowns' which in turn lead to algebraic expressions, polynomials, linear equations and their solutions. Here, students get a feel of abstract nature of mathematics for the first time. (NCERT, 2016)

Statement of Research Problem

In the Indian context, mathematics is seen as a major hurdle to cross. It is a cause of alarming number of school dropouts at secondary level (Annual Status Report on Education, 2016). In an attempt to investigate systematically the possible reasons for the 'fear of Math', algebra featured as one the most difficult to understand and hence the apprehensions about mathematics. There is a desperate need in India especially in Mathematics to explore the reasons as to why students find mathematics so difficult. To address these concerns, the

researcher framed the following research questions. Algebra in the upper primary and secondary school curriculum provides a foundation where the higher mathematics concepts rest. An understanding of why and how of the misconceptions acquired at this stage will inform teachers to better design their teaching-learning process in classroom.

Research Questions

- (i) What kind of errors and misconceptions secondary school students make when working with variables, algebraic expressions and linear equations?
- (ii) What are the implications of these acquired misconceptions in problem-solving particularly word problems?

Theoretical Framework

Since the secondary school students have already encountered algebra at upper primary stage, they are aware about the abstract nature of it. Taking cognizance of the fact that a lot of them are struggling with the arithmetic algebra transition, constructivism was taken as the most suitable framework for this study. Constructivism emphasises that concepts are formed during the learning process when students incorporate new information in their existing schema and modify it. Thus, a collection of previous knowledge, beliefs, preconceptions and misconceptions help us to look into students understanding of new knowledge. The constructivist framework asserts that students' efforts to construct knowledge may involve explaining their thinking and reasoning, which is an important part of the learning of algebraic concepts that motivated the construction of the research instruments such as the written tests.

Research Method and Procedure

A sequential exploratory design was chosen by the researcher for the study. The initial quantitative phase would aid in the selection of the students for detailed interviews in the qualitative part in the later part of the study. At the same time, the results obtained in the qualitative part would explain the why and how of the students' responses to the questions. The purpose of the research was more exploratory than descriptive, therefore mixed method research strategy was used by the researcher.

Population and Sampling

Systematic random sampling was used to draw one hundred forty-five participants from a population of two hundred twenty-three students at Kendriya Vidyalayas and Government schools at Delhi/NCR. The schools were selected because the students there had the necessary background study of algebra. All participants had passed primary school mathematics. The participants were adolescents in the 15-17 age range. English language was the medium of instruction of learning mathematics at school. The researcher selected class 10 students because it is at secondary school level of learning that students are expected to develop a strong foundation for understanding the algebraic concepts that are for studying mathematics at Senior Secondary level, or even pursue it at higher education level.

A sample is characterised by a group of subjects or people selected from the target population and has the same characteristics. For this study, class 10 students were selected

(75 from Kendriya Vidyalayas and 65 from Government schools in Delhi/NCR) using purposive sampling method. Each of the four schools allowed me to take test for the students who were free in the Zero period. This group of students made for the quantitative sample of the study. The purposive sampling technique was used to select sixteen students to be interviewed. Four students were selected from each of the four schools. However, since this paper reports only a part of the main study, the results discussed here are from the two Kendriya Vidyalayas in Delhi/NCR.

Data collection procedures

A pilot study is required to reveal any problems in the test instruments and the procedures to be used in the main study. Researcher selected the questions under the required areas of studies using the Central Board of Secondary Education (CBSE) syllabus which is used in all Kendriya Vidyalayas and most of the government schools all over India. The areas selected were algebraic expressions, variables, algebraic equations, and their implications on word problems. The researcher made a test instrument based on 8, 9 and 10 class curriculum (CBSE) followed in schools across India. The test items were based on two criteria. The first criterion was based on the conceptual understanding of the students, involving identification of patterns, relationships, and algebraic representation. Some other questions dealt with algebraic manipulations, problems involving simplification of equations, rational expressions, and word problems. The other type of questions was designed to study the use of the understanding acquired in the above concepts in problem solving. The word problems were provided in simple English. Questions included justification or reason to be provided by the students, so as to be able to gauge their logic and reasoning. Validity of the content tested was ensured by consulting the same with two experienced math teachers in each school and teacher educators. The table illustrates the categorisation of questions into the four areas of study. Table 1 shows the category of questions in the each of the four areas of study.

Table 1: Category of questions in the four areas of study

Concept	Sub-concept
Variables	<ul style="list-style-type: none"> • as unknown • as a generalised number • non variable
Algebraic Expressions	<ul style="list-style-type: none"> • simplifying expressions • equivalent algebraic expressions • comparing algebraic expressions • forming algebraic expressions
Equations	<ul style="list-style-type: none"> • simultaneous equations in different formats
Word Problems	<ul style="list-style-type: none"> • everyday Language • mathematical symbolic language

The facility value of the questions was calculated using the formula, Facility Value = $\frac{C}{N}$ where C represents students who answered the questions correctly and N represents the total number of students. Questions having a reasonable facility index were selected to be included in the main study. Too easy and too difficult questions were of no use to give any insight into the misconceptions and students' understanding, therefore, such questions were not used for the study.

Reliability

The reliability of the test instrument speaks for its worth and is an important prerequisite as it indicates, how well the test items correlate with one another. "Measurements are reliable if they reflect the true aspect and not the chance aspect of what is going to be measured" (Gilbert, 1989) The researcher used the Split Half Reliability in the study to get the reliability coefficient.

$$r_{\text{total test}} = \frac{2r_{\text{split half}}}{1 + r_{\text{split half}}}$$

After the first trial of the test, the errors were categorised and a rubric was prepared. This would give an idea of the structure of the content tested and the errors students committed due to misconceptions. The errors under the concept variables were grouped and the most commonly occurring errors were identified. The secondary school teachers at Kendriya Vidyalayas and Government schools helped in this categorisation process. With discussions and deliberations on the category of errors, a rubric was created with consensus for each of the four error categories.

Validity

The test instrument was tested for its content validity. The teachers in the four schools approved of the content as well as appropriate difficulty level. They also scrutinised the test paper with regard to the prescribed curriculum of the Central Board of Secondary Education (CBSE). The teachers also had a lot of discussions amongst themselves regarding if a particular concept was relevant to be asked, if it is there in the prescribed NCERT text books, whether it has been taught in the class or not. The test instrument was also seen and approved by two senior mathematics teachers, and two teacher education experts in the field of mathematics education.

The pilot stage was used to modify the test instruments and avoid other possible problems that might show up during the main investigation. Some of the questions were deleted because of either very high or very low values. High facility value questions indicated that most of the students could attempt it, and very low facility value indicated that very few students would even attempt it. Both the cases were not relevant for this study as the misconceptions can be studied only when a student attempts the question and struggles through problem solving. The researcher then administered the Main test to a group of 140 students. After evaluating the answer sheets thoroughly and categorising the errors in the same way as in Pilot study,

four students from each school were selected for interviews in the final study. Since this report is only a part of the main study, the interview results are not mentioned here.

Results and Discussions

Students' errors and misconceptions on variables

The results are drawn from the analysis of students' errors from the answer sheets of the Main study. The data of the test was compiled and tabulated. Focus was mainly on students' understanding of algebra, the kinds of errors, misconceptions and their origins. The twenty test items were classified into one of the four conceptual areas: variables, algebraic expressions, algebraic equations, and word problems of complexity in lingual as well as contextual levels. The errors and possible misconceptions in each question item were noted and put into various categories. However, this classification was non-exhaustive as there was disagreement among teachers on some answers given by the students. The percentage is calculated for a total of 75 students.

Table 2: Errors and possible misconceptions in each question item

Questions	Type of Misconception	Expected Answer	Incorrect Answers	Frequency of incorrect Responses	Percentage
Q1 a)	Does not identify variable	Y is a variable	M is a variable	05	7
b)	Assigning 'x' name to variables	3y is the other variable	Another variable is x	18	24
c)	Is not able to identify constant	Cost of one mango 10 is constant	Does not have a constant or C is a constant.	12	16
Q2	5x=0 Trying to make every expression an equation	5x is a generalised number	5x=0 X=0	15	20
Q3	Not familiar with product of two variables	xy means x multiplied to y	X y are two different variables which cannot be multiplied	09	12

Students see 'x' as some kind of universal variable. They use it to answer any question which is about variables and they are unsure of the answer. Though most of the students could identify the variable in this case, their wrong answers to part b) indicates that their understanding of the concept is only superficial, and most of them could only answer direct question on identification. When it came to analysis of the questions, understanding of constant was better compared to identifying a variable. The answers to Q3, related to understanding of product of variables, showed that they did not see the product of two variables feasible. Instead they wrote "xy" cannot be multiplied.

Table 3: Students' misconceptions in algebraic expressions

Questions	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q7a)	Rational expression error	1	Not possible	06	08
b)	Expression as fraction error	1	2y	07	09
c)	Zero error	0	a	05	06
d)	Factorisation error	$(x+y)(x+y)$ or x^2+y^2+2xy	$2(x+y)$	06	08
e)	making equation out of expression	$\frac{1}{4}(q+2p-24)$	$9q+2p-24=0$	25	33
f)	Error with bracket opening	$\frac{xa}{b}$	$\frac{xa}{xb}$	05	07
g)	Inappropriate cancellation ,x/x taken as 0	$\frac{a+b}{1+d}$	$\frac{a+b}{d}, 1+\frac{b}{d}$	08	11
Q6	Equivalence error in Rational Expression	$\frac{x-3}{2x} \cdot \frac{3}{2x}$	$\frac{x-3}{1}=2x$	20	27
Q8	Inappropriate cancellation due to lack of understanding of distributive law	$\frac{A(C+B)}{BC}$	$AC+A/C$	07	09
Q12	Giving values to x and comparing magnitude of denominator instead of whole fraction.	$\frac{1}{N}$	I/N is a Natural Number. It is inversely proportional	06	08
Q13 a)	Converting Expression to Equation error	$(x+y+z)$	$(x+y+z)=0$	31	41
b)	Lack of closure property for algebra letters	$7+4x$	Not Possible	32	43
c)	Like terms error	$2x+2c+5p$	$x^2+2c+5p$	05	07

Students made mistakes when they multiplied algebraic fractional expressions. For instance, for the question Simplify (ax/b) , the major error observed was that the students multiplied

both the numerator and the denominator of the fraction by the letter to get ax/bx . Sometimes they did not take cognizance of the denominator. It happens when it appears that there is no denominator. They have difficulties in realising that a single letter can be represented by an algebraic fraction with 1 as the denominator. Students think that both numerator and denominator of the fraction should be multiplied by the letter. Errors occurred when previous learning interfered in new learning. Table 3 shows the most prevalent errors among the students. These were adding unlike terms and formulating, and subsequently solving irrelevant equations. Forming of illegal equations confirms Wagner and Parker's (1984) equation-expression problem when students force expressions into equations and solve instead of simplifying. The error of adding unlike terms, that the students failed to realise that an algebraic expression $7+4x$ can be the final answer cannot be simplified. Simplify $ax+xb/x+xd$. Common incorrect answers were $a+bd$ or $a+bd$ or that emerged from processes in which the students correctly factorised out x in both numerator and denominator but failed to divide denominator and numerator by x leading to incomplete answers such as $x(a+b)/x(1+d)$ and $(a+b) \div x(1+d)$. In other solutions they just crossed out x .

Table 4: Student's Misconceptions in Solving Equations

Questions	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q16	Procedural	$X=-1$	$X=0$	12	16
Q17	Wrong operations in substitution method	$(0,4)$	$X=-4$	19	25
Q18	Added the equations most of the where subtraction was required. Ignored the denominator in $2y/3$ and added $2y/3$ and $2y$	$(4, \frac{-1}{2})$	Several wrong due to incorrect transposing and sign errors.	41	55
Q10	Did not understand structure of the subtraction statement. "Subtract" was taken as an order to minus	$10-2b$	$2b-10$	07	09
Q20	Was trying to solve the equation and not understand the balance role of "=" symbol	Balancing equation with values of m and n	$m-n=2$	29	39

Students' solution attempts to the task: Use the elimination to solve the simultaneous equations $x+y=4$; $y=2x+4$. The students' answers revealed that procedural errors occurred when students were, in the process, eliminating the unknown from the two linear equations. The students added the two equations to eliminate x instead of subtracting. This misconception is due to incomplete understanding of simplifying integers and manipulating signs. They failed to realise that they could still obtain the solutions by adding or subtracting two equations.

Table 5: Pupils' Misconceptions in Word Problems

Questions	Type of Misconception	Expected Answer	Incorrect Answer	Frequency of Incorrect answer	Percentage
Q14	*Language errors. Direct translation of Key words to symbols. *Number of times mathematical operations occurred interferes with forming expressions	$x=5$ here the emphasis was also on *the reasoning and *formulating an equation after reading the word problem. *Solving the equation errors were observed Minus sign errors occurred	15 and various incorrect answers	09	12
Q15	Linguistic errors, The relational word error. Use of two Variables C for coffee and D for Dosa.	$4d=5c$	$4c=5d$	25	33
Q19	Inability to understand relational words and hence fails to represent relationship mathematically.	$G=B+3$	$B=G+3$	28	37

The questions asked on the word problems wherein the students had to attempt problem solving were in accordance to the requirements set by NCF (2005) and National Curriculum Framework of Teacher Education NCFTE (2009). The problem-solving process in the math classroom should view students as active participants and not just recipients of knowledge. Problem solving situations provide an excellent opportunity for students to construct their knowledge and reject misconception acquired earlier, if any. This is also an opportunity to apply classroom knowledge to the real world and their immediate context. Here the word problems are treated as a subject on their own. It will also indicate how the understanding acquired in the concepts of algebraic expressions, linear equations, and variables facilitates or interferes with the ability to solve word problems. The problem-solving process here involves the following routes.

- (i) make sense of word problem
- (ii) to represent the mental diagram of the problem.
- (iii) to identify the given and the unknowns in the word problems
- (iv) to retrieve the required known knowledge for the specific word problem.
- (v) to establish a mathematical relationship between the unknowns and knowns
- (vi) to solve the mathematical equation.

(vii) to translate the mathematical variable to the original unknown.

Figure 1: Ways of solving a word problem

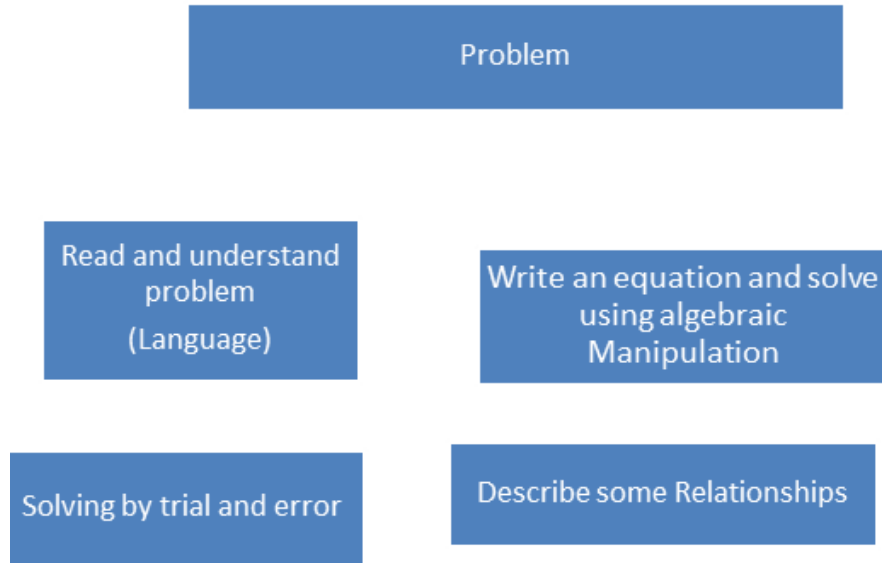


Figure 1 shows that there are different ways to solve a word problem with algebra or by simplifying through trial and error methods. A word problem is presented as a story problem in normal language. It has to be read and understood clearly first, and the given and unknowns are to be identified clearly. Therefore, using unambiguous language is as important as situating the word problem in the context of the students. Then the structure of the word problem has to be identified and the relation between the given and unknown sorted. Then follows the process of finding the solution which can use different methods as shown in the figure above. It came to light after detailed interviews that the students who were taught how to attempt and solve the story sums in different steps could successfully arrive at the solution.

Conclusion

It appears from the discussions of the students' responses that students' ability to do word problems is affected by the language they use. Indian classrooms are multicultural and students from different cultural backgrounds, and with different mother-tongues, are sharing space in the classroom. The teaching-learning process in the classroom takes place in English, which is not their first language. Therefore, the inability to comprehend a word problem at the first place demotivates them from further attempting to solve it before changing into mathematical equation or expression. For effective algebra learning, especially in Indian Classroom, the two levels of language barriers have to be addressed by the teachers in the upper primary level itself. An elaborate discussion of mathematical symbols and signs, when and where to

be used, is imperative as it makes the students comfortable to converse in mathematical language. This also adds to their confidence level and instills liking for the subject, which is a necessary requirement for effective-learning. A basic principle behind constructivist teaching learning is to understand that students' responses to the activity are meaningful to them, no matter how wrong it seems to the others. It is very important for the teacher to interpret the students' thinking and rationale behind the response and correct it in agreement with the student. So, one should not look at students' errors as road blocks, but a stepping stone to make the concept clear. Errors provide an opportunity to the teacher to look into students' thinking and plan their teaching learning to suit students' needs. This approach to errors and mistakes committed by the students will definitely replace drill and endless practice of questions with more meaningful learning.

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